Additional Notes for Exponential Random Variables

This is partially described in the book on pp. 421 - 422.

Poisson Process

The Poisson Process is an example of a Stochastic Process. Stochastic Processes are a collection of random variables that are used to represent the evolution of some random value or system over time.

The Poisson Process is a counting process. It counts the number times that some event has occurred during the interval (0,t].

The Poisson process has the following properties
1) The number of counts at t = 0 is 0.
2) The numbers of counts in nonoverlapping intervals are independent of each other.
3) The probability of exactly one count in a sufficiently small interval is the probability of one change times the length of the interval.
4) The probability of two or more counts in a sufficiently small interval is 0.

If you think back to what we were doing when we were discussing the Poisson Distribution, we were saying that if the time period changes, then we can just multiply that change of time by the rate, λ to get a new rate, λ'. in fact, we were just using the above parameters for the Poisson process to do that.

Poisson Process and Exponential Random Variables

From the book: A sequence of events that have independent Exponential waiting times between consecutive events is called a Poisson process.

Essentially:
Poisson Process: the number of successes in a specific time period.
Exponential: the time between the successes.

Therefore, you can think of the Poisson Process and an Exponential Random Variable as inverses of each other with the same parameter.

If you look at the properties of the Poisson Process, you can see now where the memoryless property of the exponential distribution comes in. This property comes about from the fact that the counts in nonoverlapping intervals are independent of each other. Therefore, it doesn't matter when we start, aka, memoryless.